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DARK MATTER AND THE EFFECTIVE VALUE OF NEWTON'S CONSTANT AT LARGE DISTANCES

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ABSTRACT

We investigate the implications of the renormalization group applied to one-loop renormalized quantum gravity for the effective value of Newton's constant on large distance scales. We find a potentially significant contribution to solving the so-called "dark matter problem."

The inflationary scenarios of the early universe solve many of the problems associated with cosmologies based on the assumptions of large scale isotropy and homogeneity. However, these successes also lead to the prediction that the density parameter $\Omega = \rho_0/\rho_c = 1$. Here $\rho_c = 3H^2/8\pi G$ is the critical energy density of the Universe, and ρ_0 is the present value of the *total* energy density, i.e., the sum of all the energy densities. This is at the basis of the so called "dark matter problem," because observations can, at most, account for a fraction of Ω.

In fact, primordial nucleosynthesis contrains the baryonic contribution to Ω to be between 0.014 and 0.16. Assuming that the virial theorem applies to groups of galaxies and to clusters of galaxies, one can estimate the contribution from these structures to Ω at the corresponding scale, with observations indicating that Ω lies between 0.1 and 0.7. The variation of Ω with distance scale is discussed in Ref. 1.

Despite the expectation that their variation with [illegible] is only logarithmic, the implicated range of distances is so wide that one must ask about the effect of quantum corrections. We know that the vacuum energy is very small but the net effect of quantum fluctuations acting over such distances may give sizeable corrections to the relevant physical quantities. In other words, it is conceivable that quantum gravity effects might play a role on the physics of the large (cosmological) scales.

We start with the asymptotically free, higher derivative gravity theory studied in detail by many authors; e.g., Refs. 2 and 3. It is described by the following four dimensional (one loop renormalizable) Euclidean action

$$S = \int d^4x\sqrt{g}\left[\Lambda - \frac{R}{16\pi G} + aW + \frac{1}{3}bR^2 + \alpha_1 R^* R^* + \kappa D^2 R\right] + S_{surface} + S_{matter}, \quad (1)$$

where Λ is the cosmological constant, G is Newton's constant, and W is related to the square of the Weyl tensor $C_{\mu\nu\rho\sigma}$ through the relationship $W = \frac{1}{2}(C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - R^* R^*)$, with $R^* \equiv \epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$. Under the requirement of aymptotically flat boundary conditions, one can compute the following renormalization group equations (RGEs) for the quantities of interest :

$$\frac{da}{dt} = \frac{133}{10} + \beta_2^{matter}, \quad \frac{db}{dt} = \frac{10}{3}\omega^2 + 5\omega + \frac{5}{12} + \beta_3^{matter}. \qquad (2a)$$

$$\frac{d}{dt}\left(-\frac{1}{G}\right) = \frac{1}{aG}\left(\frac{10}{3}\omega - \frac{13}{12\omega} + \frac{3}{2}\right) + \beta_4^{matter}, \qquad (2b)$$

where $\omega \equiv -(b/a)$, $t \equiv (32\pi^2)^{-1}log_e\mu^2/\mu_0^2)$, and μ is the renormalization scale. For Eq. (2b) we have used the gauge-invariant improvement of Ref. 3.

These renormalization group equations apply to *all* scales, and they have the property that the contributions from matter and geometry separate explicitly. We will consider the range between familiar scales and the size of the universe.

We can immediately integrate the equations for a and for ω as a function of t when $\beta_i^{matter} = 0$, since ω obeys the equation

$$\frac{d\omega}{dt} = -a(t)^{-1}\left(\frac{10}{3}\omega^2 + \frac{183}{10}\omega + \frac{5}{12}\right), \qquad (3)$$

where $a(t) = a_0 + (133/10)t$. Thus, $\omega(t) = [\omega_+ - \alpha\omega_-(a/a_0)^{-\gamma}][1 - \alpha(a/a_0)^{-\gamma}]^{-1}$, where $\omega_+ = -0.02286$ and $\omega_- = -5.467$ are the two roots of the quadratic in ω on the r.h.s. of Eq. (3). These are the two fixed points of $\omega(t)$. Also, $\alpha = (\omega_0 - \omega_+)/(\omega_0 - \omega_-)$ and $\gamma = (100/399)(\omega_+ - \omega_-) = 1.36148$. For large positive t, $\omega(t)$ approaches its UV fixed point ω_+. Similarly, at large negative t, $\omega(t)$ approaches its infrared fixed point ω_-.

If we take the ratio of Eqs. (2b) and (3) we can ultimately obtain ($\delta = 1.24253$)

$$\frac{G(t)}{G_0} = \left[\frac{\omega_0}{\omega(t)}\right]^{[illegible]}\left[\frac{\omega(t) - \omega_+}{\omega_0 - \omega_+}\right]^{[illegible]}\left[\frac{a(t)}{a_0}\right]^{\delta}. \qquad (4)$$

For $t \to +\infty$, ($\omega_0 \neq 0$) the effective Newton's constant $G(t)$ approaches zero as ω goes to ω_+, and we see explicitly that $G(t)$ is indeed asymptotically free.

In models of the type described by the action in Eq. (1), cosmological evolution leads to a scenario where there is an inflationary period which eventually settles into a Friedmann Robertson Walker era, with a matter dominated behavior that corresponds to the standard cosmology. (Note that these models assume $a = 0$.) In these models, the cosmological constant is zero, and the higher derivative terms produce inflation without a phase transition; the inflationary period flattens the metric. In the current era, the higher derivative terms are negligible. In this matter dominated era, then the matter density, ρ_m, is related to the density parameter, Ω, Hubble's constant, H, and Newton's constant by $\Omega = (8\pi/3H^2)G\rho_m$.

In our approach the invariance of the theory under choice of renormalization scale, requires that this equation be written in terms of the effective (or improved) parameters, which are solutions to the appropriate renormalization group equations. This means that instead of using a *constant* value for G or ρ_m, one must put in the solutions to the RGEs. For ρ_m and H, we will use the observed values for the physical, renormalized values. All one loop effects due to renormalization of $G(\mu)$ at present are given by Eq. (4). When this is done, we obtain $\Omega = (8\pi/3H^2)\delta(r_U, r_0)G_{lab}\rho_m$, where r_U is the size scale of the universe and $\delta(r, r_0)$ is implicitly defined by $\delta(r, r_0) = (G(r)/G(r_0))$, where r_0 is the laboratory distance scale on which G_{lab} is measured. This ratio is then determined by Eq. (4). If we take G_0 as G_{lab}, this requires that $t = (32\pi^2)^{-1}ln(r_0^2/r^2)$. Because of the logarithm, this identification is correct up to next order in the loop expansion of quantum gravity. The only difference from more familiar applications of the renormalization group is that, now, laboratory scales are considered the short distance regime, compared to the larger scale distances of the Universe. Thus, it is the long distance, negative t regime of "infrared confinement" that is of interest here.

The function $\delta(r, r_0)$ is a growing function of r, so that for a given physical energy density, the present value of Ω follows $\delta(r, r_0)$. However, if the present calculation were to account for the large scale missing dark matter of the universe, G(r) would vary a number of percent over the size of the solar system. Even so, it is clear that values of ρ_m *below* the conventional critical value could at least partially account for the observed deceleration of the expansion of the universe. It is also clear that what has conventionally been interpreted as "dark matter" need not be present at all, if further and more detailed calculations allow the variation of G over the solar system to be small.

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